INSTITUTE AND FACULTY OF ACTUARIES

BACKGROUND READING

Subject CA3 – Communications

(Written)

Scenario: XYZ Pension Scheme

Note to students

The information in this background reading document was provided to candidates during the day of the examination in advance of the examination itself. Under the current version of CA3 all the background information is included in the question paper.

You have recently joined a firm of consulting actuaries advising pension schemes on funding matters. You have been assigned to the team looking after the XYZ pension scheme and you report directly to your manager who is the actuary in charge of the client account. Your team has just completed the valuation of the XYZ pension scheme and produced a draft report showing the results, including the balance sheet of assets and liabilities, which your manager has recently sent to the Chairman of the Scheme's Trustees.

Assumptions have been adopted for the following key financial and demographic parameters:

- Investment Return (% per annum)
- Inflation (% per annum)
- Mortality – post retirement (used standard mortality tables for pensioner lives with an adjustment allowing for the scheme's experience and allowance for future mortality improvements)

Other financial and demographic assumptions adopted for the valuation include:

- Mortality – pre retirement (used standard mortality tables for insured lives with an adjustment allowing for the scheme's experience and allowance for future mortality improvements)
- Promotional salary increases (age-dependent rates were derived and are based on discussions with the sponsoring employer. Different rates apply to different staff sections, for example factory workers are on a different salary scale to executives).
- Withdrawal rates (based on the scheme's experience, age-dependent rates of scheme members leaving the scheme and/or employment with the sponsoring employer).
- Early Retirement rates in normal health (based on the scheme's experience, allowance for early retirement from age 60 up to the Normal Retirement Age of 65).
- Early Retirement rates on the grounds of ill health – no allowance made for this valuation.
- Late Retirement beyond age 65 – no allowance made for this valuation.
- Allowance for conversion of pension to cash on retirement – no allowance made for this valuation.
- Allowance for spouse's benefits – a fixed percentage of scheme members are assumed to be married.
- Spouse's mortality (based on pensioner mortality tables, assuming wives are three years younger than husbands, on average).
- Expenses – 5% loading applied to liabilities to allow for scheme expenses.

Your manager has been on the telephone to the Chairman of the Trustees of the XYZ Pension Scheme as a courtesy call to check that the report arrived safely.

The following material is an extract from the 2011 Core Reading for CA1:

UNIT 12 — MODELLING

Syllabus objectives

7.1.1 Describe the approaches available to produce the solutions.

7.1.2 Describe the use of actuarial models to support the methodology used in terms of:

- the objectives of and requirements for building a model
- the basic features of a model required to project future cash and revenue flows
- the use of these models for:
  - pricing or setting future financing strategies
  - risk management
  - assessing the capital requirements and the return on capital or the funding levels required
  - assessing the provisions needed for existing commitments to provide benefits on contingent events
  - pricing and valuing options and guarantees
- how sensitivity analysis of the results of the models can be used to help decision making

1 Possible approaches to solving actuarial problems

There are various approaches that can be taken to produce the solution to an actuarial problem. Simple problems can have a simple solution that is arrived at by some straightforward mathematics, for example calculating the yield on a fixed interest asset, or the present value of a series of known cash flows.

However the majority of problems that require actuarial skills involve taking a view on uncertain future events. It is possible to take a view on various parameters, such as future economic conditions, future mortality rates, or the amount of business that a provider might write in future, and produce a single answer that is appropriate in these best estimate conditions. If this is done then the communication of the solution to the client needs particular care, because of the uncertainties in the underlying assumptions.

In these circumstances the client is likely to wish to know the variability of the answer provided, should circumstances not be as estimated. To assess the effects of varying the assumptions used in producing the answer, it is normally necessary to use an actuarial model of future events.

A model can be defined as "a cut-down, simplified version of reality that captures the essential features of a problem and aids understanding". The final phrase in this definition recognises the importance of being able to communicate the results effectively. Modelling requires a balance to be struck between realism (and hence complexity) and simplicity (for ease of application, verification and interpretation of results).

When faced with an actuarial problem, there are various approaches to modelling:

- a commercial modelling product could be purchased
- an existing model could be reused, possibly after modification
- a new model could be developed

The merits of each of these approaches will depend on amongst other things:

- the level of accuracy required
- the "in-house" expertise available
- the number of times the model is to be used
- the desired flexibility of the model
- the cost of each option

Other issues include making sure that the model used is fit for the purpose for which it is being used. This is particularly relevant when a model is being purchased from an external provider or when an existing model is being reused for a different purpose.

There are now a large number of stochastic asset models in existence, in both the public and private domains. There are fewer models available for other variables, such as mortality and voluntary discontinuance, but these are starting to be developed.

The prime objective in building a model is to enable a provider of financial products to be run in a sound financial way. An actuary will normally be giving advice to the provider to achieve this aim. To this end, models will be used to assist in the day-to-day work of the provider and to provide checks and controls on its business.

2 Objectives of and requirements for building a model

Models will need to satisfy the following requirements:

- The model being used must be valid, rigorous enough for its purpose and adequately documented.

- The model chosen should be capable of reflecting the risk profile of the financial products, schemes, contracts or transactions being modelled adequately.

- The parameters used must allow for all those features of the business being modelled that could significantly affect the advice being given.

- The inputs to the parameter values should be appropriate to the business being modelled and take into account any special features of the provider and the economic and business environment in which it is operating.

- The workings of the model should be easy to appreciate and communicate. The results should be displayed clearly. The model should exhibit sensible joint behaviour of model variables.

- The outputs from the model should be capable of independent verification for reasonableness and should be communicable to those to whom advice will be given.

- The model, however, must not be overly complex so that either the results become difficult to interpret and communicate or the model becomes too long or expensive to run, unless this is required by the purpose of the model. It is important to avoid the impression that everything can be modelled.

- The model should be capable of development and refinement — nothing complex can be successfully designed and built in a single attempt.

- A range of methods of implementation should be available to facilitate testing, parameterisation and focus of results.

It will be necessary to decide between deterministic and stochastic modelling processes. A deterministic model is more readily explicable to a non-technical audience, since the concept of variables as probability distributions is not easy to understand. It is clearer what economic scenarios have been tested. The model is usually easier to design and quicker to run. The disadvantage is that it requires thought as to the range of economic scenarios that should be tested.

A stochastic model tests a wider range of economic scenarios. The programming is more complex and the run time longer, but the benefit is in the quality of the result. It does depend on the parameters that are used in any standard investment model. Stochastic models are particularly important in assessing the impact of financial guarantees.

In many cases the problem can be solved by a combination of stochastic and deterministic modelling. Variables whose performance is unknown and where the risk associated with them is high might be modelled stochastically, while other variables can sensibly be modelled deterministically. For example a model for pricing an investment guarantee attached to a life insurance policy might use a stochastic investment model, but would be unlikely to model fluctuations in mortality rates other than deterministically. This is because it is normally self-evident which direction of movement in mortality rates would give rise to financial difficulties.

In all cases the dynamism of the model is vital. Rules need to be determined as to how the various features would interact in different circumstances. For example how life assurance bonus rates would vary with fixed interest yields or how unemployment rates would vary with economic conditions. These interactions are usually much more important than the type of model.

Considerable actuarial judgement may be required in choosing and using the model and in setting the parameters and interactions between the different features.

3 Basic features of a model required to project future cash and revenue flows

An actuarial model needs to allow for all the cash flows that may arise. These will depend on the nature of the financial products, schemes, contracts or transactions being modelled and any discretionary benefits they carry.

It also needs to allow, where appropriate, for the cash flows arising from any supervisory or commercial requirement to hold reserves and to maintain adequate solvency capital.

The cash flows need to allow for any interactions, particularly where the assets and the liabilities are being modelled together.

Where the business being modelled includes options, for example an option to take out a new term assurance contract without providing further evidence of health, the potential cash flows from such options and the take-up rate need to be allowed for.

In some cases there is a need to use stochastic models and simulation, for example, when assessing the impact of financial guarantees or to allow for investment mismatching risks.

The time period for calculating the cash flows in the projection needs to be chosen bearing in mind that:

- The more frequently the cash flows are calculated the more reliable the output from the model, although there is a danger of spurious accuracy.
- The less frequently the cash flows are calculated the faster the model can be run and results obtained.

Deterministic modelling could involve the following steps:

- Specify the purpose of the investigation.
- Collect, group and modify data.
- Choose the form of the model, identifying its parameters and variables.
- Ascribe values to the parameters using past experience and appropriate estimation techniques.
- Construct a model based on the expected cash flows.
- Check that the goodness of fit is acceptable. This can be done by running a past year and comparing the model with the actual results.
- Attempt to fit a different model if the first choice does not fit well.
- Run the model using selected values of the variables.
- Run the model using estimates of the values of variables in the future.
- Run the model several times to assess the sensitivity of the results to different parameter values

Stochastic modelling could involve the following steps:

- Specify the purpose of the investigation.
- Collect, group and modify data.
- Choose a suitable density function for each of the variables to be modelled stochastically.
- Specify correlation between variables.
- Construct a model based on the expected cash flows.

- Check the goodness of fit is acceptable. This can be done by running a past year and comparing the model with the actual results.

- Attempt to fit a different model if the first model does not fit well.

- Run the model many times, each time using a random sample from the chosen density function(s).

- Produce a summary of the results that shows the distribution of the modelled results after many simulations have been run.

Stochastic and deterministic approaches can be combined in a single model in various ways. For example in a model to assess the likely variability in general insurance claims experience, there are three ways in which a model may be applied:

- Determine the number of claims stochastically and associate with a deterministic mean claim cost. Ideally the claim numbers would be divided into various homogeneous groups in terms of claim size.

- Determine the claim amounts stochastically for the expected number of claims.

- Determine both claim amounts and numbers stochastically, using a collective risk model.

4 Use of models

4.1 Pricing and setting future financing strategies

A model could be developed to determine a premium or charging structure for a new or existing product that will meet a life insurance company's profit requirement.

The underlying business being modelled will typically comprise a very wide range of different policies, and these will need to be brought together into a manageable number of relatively homogeneous groups. The groupings need to be made in a way that each policy in a group is expected to produce similar results when the model is run. It is then sufficient for a representative single policy in each group to be run through the model, the result to be found, and for this result to be scaled up in order to give the result of the total set of policies in the group. The representative single policy in a group is termed a "model point" and a number of such "model points" can then be used to represent the whole of the underlying business.

A number of model points will be chosen to represent the expected new business under the product. In the case of an existing product, the profile of the existing business, modified to allow for any expected changes in future, can be used to obtain the model points. For a new product, the profile of any similar existing product combined with advice from the company's marketing department would be used.

For each model point, cash flows would be projected, allowing for reserving and solvency margin requirements, on the basis of a set of base values for the parameters in the model. The net projected cash flows will then be discounted at a rate of interest, the risk discount rate:

This could be a rate that allows for:

- the return required by the company; and

- the level of statistical risk attaching to the cash flows under the particular contract, i.e. their variation about the mean as represented by the cash flows themselves

The level of statistical risk could be assessed:

- In some situations analytically, by considering the variances of the individual parameter values used.

- By using sensitivity analysis, as described below, with deterministically assessed variations in the parameter values.

- By using stochastic models for some or all of the parameter values and simulation.

Alternatively a stochastic discount rate could be used.

In theory, a separate risk discount rate should be applied to each separate component of the cash flows, as the statistical risk associated with each component will be different. In practice a single risk discount rate is commonly used, bearing in mind the "average" risk of the product.

The premium or charges for the model point can then be set so as to produce the profit required by the company.

The premiums, or charges, produced need to be considered for marketability. This might lead to a reconsideration of:

- The design of the product, so as either to remove features that increase the risks within the net cash flows, or to include features that will differentiate the product from those of competing companies.

- The distribution channel to be used, if that would permit either a revision of the assumptions to be used in the model, or a higher premium or charges to be used without loss of marketability.

- The company's profit requirement.

- The size of the market.

- Whether to proceed with marketing the product.

The net cash flows in respect of the model points, appropriately scaled up for the expected new business under the product, will be incorporated into a model of the business of the whole company. It is possible for the desired level of profitability to be reached in aggregate, without requiring every single model point to be profitable in its own right. If certain model points are unprofitable the aggregate profitability of the business is then exposed to changes in mix and volume of the contracts sold.

The actuary can assess the impact on capital management of writing the product, by observing the modelled amount and timing of cashflows. If capital is a problem, this may lead to a reconsideration of the design of the product so as to reduce or amend the timing of its financing requirement.

Once acceptable premiums or charges, have been determined for the model points, premiums or charges for all contract variations can be determined.

For a benefit scheme, the equivalent to determining the price for a product is setting the future financing strategy, and similar modelling techniques can be used. The existing

membership can be divided into categories and represented by a number of model points. Similar potential new members can be represented, perhaps by a single model point at the average entry age and salary.

A potential financing strategy is determined, in terms of both the amount and timing of future contributions. The cash flows from the existing assets and future contributions can be modelled, as can the liability cash flows, taking all the possible decrements into account.

Unlike an insurance company, a benefit scheme can show a deficit at a point in time — that is the value of accumulated assets does not exceed the value of accrued liabilities, provided that there is a sponsor with a good enough covenant to make good the shortfall. However the scheme does need to be solvent to the extent that it has sufficient assets to meet benefit outgo as it falls due. A well designed model will check this feature as well as determining the discounted value of asset and liability cash flows.

Considerations such as the choice of risk discount rate, and the need to test sensitivities to changes in conditions are all similar to those in product pricing.

4.2 Risk management

Cash flow models are used in risk management to determine the amount of capital that it is necessary to hold to support the risks retained by a financial institution. The various modelling approaches are discussed in Unit 11, section 5.

As well as the full corporate model to assess capital requirements, models of specific risks can be used to determine the extent of a risk event that will occur at a given probability, even if a full stochastic model is too slow, too complex, or otherwise not used. For example a company that is targeting being able to withstand a 0.1% probability of ruin needs to know what equity market fall to test in a deterministic scenario.

A standard equity market stochastic model can be used and calibrated to historic performance of the market being considered. By running the model several thousand times and ranking the results, the equity fall that gives the one in a thousand worst result can be found.

4.3 Assessing the capital requirements and the return on capital or the funding level required

The net cash flows for the model points described in the section on pricing above can be grossed up for the expected new business and used to assess the amount of capital that will be required to write the product, either on a regulatory or an economic basis. To this can be added any one-off development costs to the extent that they are not amortised and included in the cash flows used. This gives the total capital requirement and can be compared with the profits expected to emerge from the product so as to determine the expected return on that capital.

4.4 Assessing the provisions needed for existing commitments to provide benefits on future financial events

The normal procedure for determining life assurance or pension scheme liabilities is to value the benefits for each policy or scheme member individually. In many territories this may be required by legislation or regulation. Consequently for published results there is little scope for using model points. However, before finalising a published basis, many "what if" questions might be asked. These could be answered by running a model of the business. For smaller schemes or sections of a company's business it might be just as

quick to run the whole business to answer the question and eliminate the model risk, given the current speed of computers.

As part of assessing a realistic provision it is necessary to consider the effect of changes in economic scenarios. For example, using a stochastic model of possible asset movements, a provision that would be adequate in all but a small proportion of scenarios can be determined.

4.5 Pricing and valuing options and guarantees

In most cases the options and guarantees that give a provider of benefits on future financial events cause for concern are those that are dependent on future investment returns, or an investment value (yield or capital value) at some future point in time. Because of the uncertainty, a stochastic investment model should be used to assess the provisions necessary for such guarantees.

If future returns exceed a certain level, or if a value or index is above (or below) a fixed value at some future point, there will be no cost to the company. But if they are below that level, there will be a cost, which increases as returns reduce. Hence a range of future investment scenarios should be tested.

Example

A unit-linked life assurance policy guarantees to pay a maturity value equal to the sum of premiums on the chosen maturity date, or the value of units allocated if greater. At all other times the surrender value is based on the value of units.

Discuss the steps involved in assessing the provision to be made for the cost of this guarantee.

Solution

The steps involved are:

- Choose a stochastic asset model — a complex model gives better results but takes longer to run.
- Determine assumptions — particularly unit growth rate mean and volatility.
- Determine consistent deterministic assumptions for mortality and surrender rates and future expenses.
- Consider dynamic links between assumptions — e.g. lapse rates to unit values.
- Choose a time period — probably annual for efficient running.
- Determine appropriate model points for the portfolio.
- The model will project the unit values to maturity, allowing for future premiums and all decrements.
- This will be done for a large number of randomly generated investment scenarios — say between 1,000 and 5,000.
- For each scenario and each model point, the projected unit value will be compared with the guaranteed maturity value, and the cost for that particular scenario and model point determined.

- The projected costs are discounted to the present, scaled up by the appropriate factors, and summed across all model points.

- The average across all scenarios is the expected cost of the guarantee

- The variability should be assessed by looking at the quartiles and $5^{th}/95^{th}$ percentiles, when the results are ranked.

- For reserving purposes an appropriate ruin probability needs to be chosen. Perhaps 1 in 100, in which case the reserve is the 99^{th} percentile.

5 Sensitivity analysis

The results from the models depend on the model itself and the values assigned to the parameters in the model. Models should not be treated as black boxes the output of which is assumed to be correct.

The use of a stochastic model goes some way to illustrating the potential variability of the experience, but the results that it produces are still dependent on the accuracy of the model and its parameter values. In the case of a deterministic model, the potential uncertainty of the results is greater, because fewer scenarios are tested.

The re-running of a model (deterministic or stochastic) with different, but feasible, parameter values will produce alternative results and hence help to illustrate the potential deviations. The re-running with a series of different sets of parameter values, perhaps chosen from a probability distribution for such values, will help to illustrate the likely range in which actual experience may lie, perhaps as far as creating a probability distribution for this experience.

For example, consideration of the effect of a change in the membership profile of a funded pension scheme may be needed to illustrate the extent of potential variability in future contributions if the model used is based on a stable membership profile.

There is the possibility of model error if the model developed is not appropriate for the financial products, schemes, contracts or transactions being modelled. Checks of goodness of fit will be needed to assess the suitability of the model.

The effect of mis-estimation of parameter values can also be investigated by carrying out a sensitivity analysis. This involves assessing the effect on the output of the model of varying each of the parameter values. When doing this any correlation between different parameters should be allowed for.

In the case of a model used for pricing, the results from the sensitivity analysis will help to assess the margins that need to be incorporated into the parameter values. In the case of models used to assess return on capital and profitability of existing business, the results will enable the actuary to quantify the effect of departures from the chosen parameter values when presenting the results of the model to the company.

The statistical risk associated with the parameter values can be allowed through the risk element of the risk discount rate. An alternative would be to use a predetermined discount rate and then assess the effect on the results of the models of statistical risk.

Where a probability distribution can be assigned to a parameter, it may be possible to derive the variance of the profit or return on capital analytically. More generally, a sensitivity analysis, as described above, can be carried out. Whichever of these two is

used, they will again help in assessing margins or in quantifying the effect of departures from the chosen parameter values when presenting the results of the model.

END OF BACKGROUND READING