INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

27 April 2015 (pm)

Subject CT5 – Contingencies
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 14 questions, beginning your answer to each question on a new page.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Calculate $A_{50:\overline{4}|}$.

Basis:

Mortality $q_{50} = 0.05$
$q_{51} = 0.06$
$q_{51+t} = 1.1q_{50+t}$ for $t \geq 1$

Interest 6% per annum [3]

2 Describe how the quality of housing affects mortality and morbidity. [3]

3 Suppose α and β are the only two independent modes of decrement and $\mu_x^{\beta} = \frac{1}{4}\mu_x^{\alpha}$

Express $(aq)_x$ in terms of μ_x^{β}. [3]

4 Calculate:

(a) ${}_{10|15}q_{60}$

(b) ${}_{12}p_{[50]+1}$

(c) $a^{(4)}_{40:\overline{10}|}$

Basis:

Mortality AM92
Interest 6% per annum [4]

5 Draw a multiple state model diagram for the parameters involved using the service table from the PEN tables on Page 142 of the Formulae and Tables for Examinations and labelling your diagram clearly. [5]

6 A life aged 55 exact purchases a 3-year term assurance with sum assured of £150,000 payable if death occurs during the term of the policy. Level premiums of £900 are payable annually in advance throughout the term of the policy or until earlier death. The death benefit is payable at the end of the policy year of death.

Calculate the expected present value of the profit or loss to the office on the contract.

Basis:

Mortality	AM92 Select
Interest	3% per annum
Initial expenses	£260
Renewal expenses	£70 per annum incurred at the start of both the second and third policy year

Assume no reserves are required for this policy. [6]

7 Calculate ${}_{1.75}p_{82.75}$.

(a) Using the method of Uniform Distribution of Deaths.

(b) Using the method of Constant Force of Mortality.

Basis:

Mortality ELT15(Males) [6]

8 (a) Explain why lives are subdivided into separate groups for the analysis of mortality.

(b) Suggest three types of selection with an example for each. [7]

9 On 1 January 1999, an insurance company issued a without profit whole life policy to a life aged 45 exact. The sum assured on the policy is £125,000 which is payable at the end of the year of death. Level premiums are payable annually in advance to age 65 or until earlier death. The company calculated the premium on the following basis:

Mortality	AM92 Select
Interest	6% per annum
Initial expenses	75% of the first year's premium, incurred at outset
Renewal expenses	5% of the second and each subsequent year's premium, incurred at the beginning of the respective policy years
Claims expense	£325 payable at the end of the year of death

(i) Show that the annual premium is approximately £1,883. [4]

On 31 December 2013, immediately before the premium then due, the life wishes to surrender the policy. The insurance company calculates a surrender value equal to the gross prospective policy reserve, using the following basis:

Mortality	AM92 Ultimate
Interest	6% per annum
Expenses	Ignore

(ii) Calculate the surrender value payable by the insurance company. [3]

[Total 7]

10 (i) Calculate: $\overline{A}^{\,1}_{40:50}$. [2]

Basis:

Mortality	$\mu_x = 0.04$ throughout life for the life aged 40
	$\mu_x = 0.06$ throughout life for the life aged 50
Rate of interest	5% per annum

Two lives aged 40 and 50 exact purchase a policy with the benefit in part (i) above and a sum assured of 75,000. The benefit is funded by a premium payable continuously for a 30-year period or until the first death if earlier. The premium is paid at a level annual rate for the first 20 years, then reduces by 25% to be paid at the lower level annual rate for the remainder of the period.

(ii) Calculate the initial level annual premium using the basis in part (i) above. [6]

[Total 8]

11 A special joint life annuity is issued to a male life now aged 65 exact and a female life now aged 62 exact. The annuity is payable monthly in arrear and is subject to the following conditions:

- The amount of the annuity while both lives survive is 100,000 per annum.
- If the male life dies first leaving the female life surviving the annuity reduces to 50,000 per annum payable until she dies.
- If the female life dies first leaving the male life surviving the annuity reduces to 75,000 per annum payable until he dies.
- In addition if either life is alive at the 10^{th} and 20^{th} anniversaries of the policy a cash lump sum of 20,000 is paid at each date.

Calculate the expected present value of the annuity.

Basis:

Mortality	PMA92C20 and PFA92C20
Interest	4% per annum
Expenses	Nil

[9]

12 An insurance company issues a 25-year with-profit endowment assurance policy to a life aged 40 exact. The sum assured of £75,000 plus declared reversionary bonuses are payable on survival to the end of the term or immediately on death if earlier.

The insurance company assumes that future reversionary bonuses will be declared at a rate of 3% of the sum assured, simple and vesting at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year of death). Premiums are payable in advance throughout the term of the policy or until earlier death.

Calculate the monthly premium.

Basis:

Mortality	AM92 Select
Interest	6% per annum
Initial commission	115% of the first monthly premium
Initial expenses	£210
Renewal commission	2.5% of each monthly premium payable excluding the first
Renewal expenses	£85 per annum, inflating at 1.92308% per annum compound, at the start of the second and subsequent policy years.
Inflation	For renewal expenses, the amount quoted is at outset, and the increases due to inflation start immediately.

[9]

13 On 1 January 2004, an insurance company issued 15-year temporary assurance policies to 3,000 lives then aged 45 exact. For each policy, the sum assured is £100,000 for the first 10 years, and £40,000 thereafter. The sum assured is payable immediately on death and level annual premiums are payable in advance throughout the term or until earlier death.

Basis:

Mortality	AM92 Ultimate
Interest	4% per annum

(i) Show that the annual premium payable for each policy is approximately £233 using the basis above. [4]

(ii) Calculate the reserve per policy as at 31 December 2013, assuming the reserving basis is the same as the premium basis. [3]

(iii) (a) Describe the disadvantages to the insurance company of issuing this policy.

(b) Suggest two examples of how the terms of the policy could be altered so as to remove these disadvantages. [4]

There were 122 deaths between 2004 and 2012 inclusive and a further 12 deaths in 2013.

(iv) Calculate the mortality profit or loss to the insurance company in 2013 on the basis above. [2]

[Total 13]

14 A life insurance company issues a three-year unit-linked endowment assurance policy to a life aged 58 exact under which level premiums of £3,000 are payable annually in advance throughout the term of the policy or until earlier death. The premium allocation rate (%) at time t is given by:

$[75 + 20t]$ where $t = 0$, 1 and 2.

The units are subject to a bid-offer spread of 5%. An annual management charge of 0.75% of the bid value of units is deducted at the end of each policy year.

Management charges are deducted from the unit fund before any death, surrender or maturity benefits are paid.

If the policyholder dies during the term of the policy, the death benefit of £9,000 or the bid value of the units if higher, is payable at the end of the policy year of death. The policyholder may surrender the policy only at the end of each policy year. On surrender at the end of the policy year or on survival to the end of the term, the current bid value of the units is payable.

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund	4% per annum
Rate of interest on non-unit fund cash flows	2% per annum
Mortality	AM92 Select
Surrender	10% at the end of first, second and third policy years only
Initial expenses	£275
Renewal expenses	£70 per annum on the second and subsequent premium dates
Initial commission	5% of first premium
Renewal commission	2% of the second and subsequent years' premiums
Risk discount rate	6% per annum

(i) Calculate the profit margin for the policy on the assumption that the company does not zeroise future expected negative cash flows. [13]

Suppose the company sets up reserves in order to zeroise future negative expected cash flows.

(ii) Calculate the profit margin for the policy allowing for the cost of setting up these reserves. [4]

[Total 17]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2015 examinations

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context at the date the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

F Layton
Chairman of the Board of Examiners

June 2015

General comments on Subject CT5

CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions actuarial work.

Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given where appropriate to different valid points made which do not appear in the solutions below.

In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

Comments on the April 2015 paper

The general performance was higher than usual this session compared to previous ones although it was felt that this paper was roughly of the same standard as previous ones. Questions that were done less well were Q1, Q10 part (ii), Q11, Q13 part (ii) and Q13 part (iv) and Q14 part (ii). The examiners hope that the detailed solutions given below will assist students with further revision.

However most of the short questions were very straightforward where an answer could be produced quickly and this is where many successful candidates scored particularly well. Students should note that for long questions reasonable credit is given if they can describe the right procedures although to score high marks reasonable accurate numerical calculation is necessary.

1

$$\ddot{a}_{50:\overline{4}|} = 1 + \frac{(1-.05)}{1.06} + \frac{(1-.05)(1-.06)}{(1.06)^2} + \frac{(1-.05)(1-.06)(1-.06(1.1))}{(1.06)^3}$$

$$= 1 + 0.89623 + 0.79477 + 0.70029 = 3.39129$$

$$A_{50:\overline{4}|} = 1 - d(6\%)\ddot{a}_{50:\overline{4}|} = 1 - \frac{.06}{1.06}(3.39129) = 0.80804$$

This question gave many students difficulties. The answer was most easily obtained quickly using premium conversion formulae as above. The alternative method of direct computation is, of course, possible but is more involved.

2 The standard of housing encompasses not only all aspects of the physical quality of housing (e.g. state of repair, type of construction, heating, sanitation) but also the way in which the housing is used e.g. overcrowding and shared cooking.

These factors have an important influence on morbidity, particularly that related to infectious diseases (e.g. from tuberculosis and cholera to colds and coughs) and thus on mortality in the longer term.

The effect of poor housing is often confounded with the general effects of poverty.

A straightforward bookwork question generally well answered. The main omission by students was the comment in the 3rd paragraph.

3

$$(aq)_x^{\alpha} = \frac{\mu_x^{\alpha}}{\mu_x^{\alpha} + \mu_x^{\beta}}\left(1 - e^{-(\mu_x^{\alpha} + \mu_x^{\beta})}\right) \text{ and}$$

$$(aq)_x^{\beta} = \frac{\mu_x^{\beta}}{\mu_x^{\alpha} + \mu_x^{\beta}}\left(1 - e^{-(\mu_x^{\alpha} + \mu_x^{\beta})}\right)$$

$$\text{Thus } (aq)_x = (aq)_x^{\alpha} + (aq)_x^{\beta} = \left(1 - e^{-5\mu_x^{\beta}}\right)$$

Question was generally well done. Students who left the final answer in integral form also received full credit.

4 (a) $${}_{10|15}q_{60} = \frac{l_{70} - l_{85}}{l_{60}} = \frac{8054.0544 - 3385.2479}{9287.2164} = 0.50271$$

(b) $${}_{12}p_{[50]+1} = \frac{l_{63}}{l_{[50]+1}} = \frac{9037.3973}{9686.9669} = 0.93294$$

(c) $$a^{(4)}_{40:\overline{10}|} = a^{(4)}_{40} - \frac{v^{10} l_{50}}{l_{40}} a^{(4)}_{50} \text{ at } 6\%$$

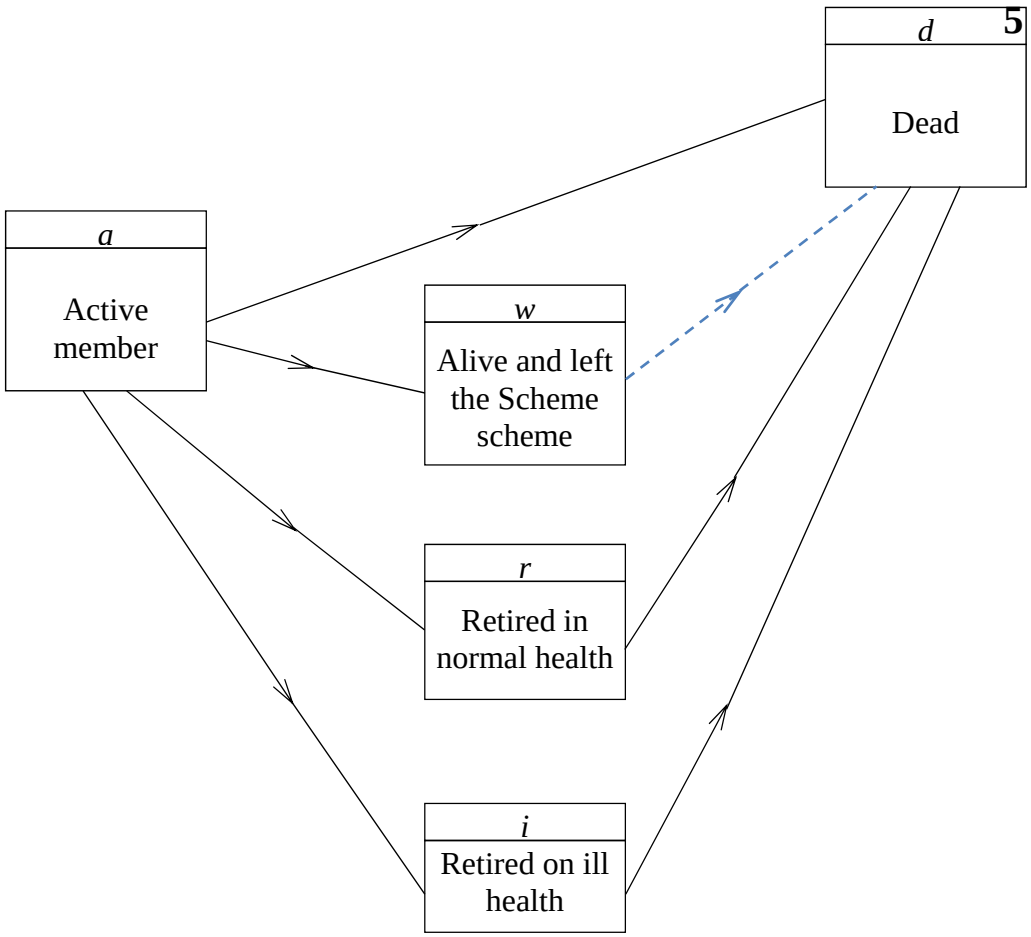
$$= \left(\ddot{a}_{40} - \frac{5}{8}\right) - \frac{v^{10} l_{50}}{l_{40}} \left(\ddot{a}_{50} - \frac{5}{8}\right)$$

$$= (15.491 - 0.625) - \frac{0.55839 \times 9712.0728}{9856.2863}(14.044 - 0.625)$$

$$= 14.866 - 7.383 = 7.483$$

Parts (a) and (b) were straightforward and well done. Many students in (c) did not obtain the correct relationship for the adue function in line 2 of the formulae above.

5



Straightforward question generally well done. Note there is in reality no connection from Withdrawn to Dead as this is not a feature of the PEN tables and lives have left the scheme experience altogether. Also there are no probabilities shown for the PEN tables for states w, r and i to d so students who did not include these in the diagram were given full credit.

6

	$q_{[x]}$	$q_{[x-1]+1}$	$q_{[x-2]+2}$
55	**0.003358**		
56		**0.004903**	
57			**0.005650**

EPV premiums $= 900\{1 + v^*p_{[55]} + v^{2*}\, p_{[55]}{}^*p_{[55]+1}\}$
$= 900\{1 + v^*(1 - 0.003358) + v^{2*}(1 - 0.003358)^*(1 - 0.004903)\}$
$= 900(1 + 0.96761 + 0.93482) = 2612.19$

EPV benefits $= 150{,}000\{v^*q_{[55]} + v^{2*}p_{[55]}{}^*q_{[55]+1} + v^{3*}p_{[55]}{}^*p_{[55]+1}{}^*q_{57}\}$
$= 150{,}000\ \{0.0032602 + 0.0046060 + 0.0051279\} = 1949.12$

EPV expenses $= 260 + 70\{v^*p_{[55]} + v^{2*}\, p_{[55]}{}^*p_{[55]+1}\}$
$= 260 + 70\{v^*(1 - 0.003358) + v^{2*}(1 - 0.003358)^*(1 - 0.004903)\}$
$= 260 + 70(0.96761 + 0.93482) = 393.17$

EPV profit $= 2612.19 - 1949.12 - 393.17 = 269.90$

Alternatively, using cash flow approach:

Yr	*premium*	*expense*	*interest*	*claim*	*profit vector*	*cumulative probability of survival*	*discount factor*	*net present value*
1	900.00	260.00	19.20	503.70	155.50	1.000000	.97087	150.97
2	900.00	70.00	24.90	735.45	119.45	0.996642	.94260	112.22
3	900.00	70.00	24.90	847.50	7.40	0.991755	.91514	6.72

Total net present value of profit = 269.91

This question was generally well done by well prepared students.

7 (a)

$$
\begin{aligned}
{}_{1.75}p_{82.75} &= {}_{0.25}p_{82.75} \times p_{83} \times {}_{0.5}p_{84} \\
&= (1 - {}_{0.25}q_{82.75})(1 - q_{83})(1 - {}_{0.5}q_{84}) \\
&= \left(1 - \frac{0.25q_{82}}{(1 - 0.75q_{82})}\right)(1 - q_{83})(1 - 0.5q_{84}) \\
&= \left(1 - \frac{0.25 \times 0.11279}{(1 - 0.75 \times 0.11279)}\right) \times (1 - 0.12235) \times (1 - 0.5 \times 0.13270) \\
&= 0.79418
\end{aligned}
$$

(b)

$$
\begin{aligned}
{}_{1.75}p_{82.75} &= {}_{0.25}p_{82.75} \times p_{83} \times {}_{0.5}p_{84} \\
&= (p_{82})^{0.25} \times p_{83} \times (p_{84})^{0.5} \\
&= (1 - 0.11279)^{0.25} \times (1 - 0.12235) \times (1 - 0.13270)^{0.5} \\
&= 0.79325
\end{aligned}
$$

This question was generally well done.

8 (a) When a life table is constructed it is assumed to reflect the mortality experience of a homogeneous group of lives. This table can then be used to model the experience of a homogeneous group of lives which is suspected to have a similar experience.

If a table is constructed for heterogeneous group then the mortality experience will depend on the exact mixture of lives with different experiences used to construct the table. Such a table could only be used to model mortality in a group with the same mixture.

For this reason separate mortality tables are usually constructed for groups which are expected to be heterogeneous.

(b) Choose from:

- Full choice available here from
- Temporary Initial Selection
- Class Selection
- Adverse Selection
- Time Selection
- Spurious Selection

A straight bookwork question generally well done.

9 (i) Let P be the annual premium for the contract. Then:

EPV of premiums is:

$$P\ddot{a}_{[45]:\overline{20}|} = 11.888P$$

EPV of benefits and claim expense:

$$125,325A_{[45]} = 125,325 \times 0.15918 = 19,949.23$$

EPV of other expenses:

$$0.75P + 0.05P\left[\ddot{a}_{[45]:\overline{20}|} - 1\right] = 1.2944P$$

Equation of value gives

$$11.888P = 19,949.23 + 1.2944P$$

$$\Rightarrow P = \frac{19,949.23}{10.5936} = £1,883.14$$

(ii) gross prospective reserve

$$= 125,000A_{60} - 1883.14\ddot{a}_{60:\overline{5}|} = 125,000 \times 0.32692 - 1883.14 \times 4.39$$
$$= 40,865.0 - 8,266.98 = 32,598.02$$

Generally well done. The main omission that some students counted the claim expense within gross prospective reserve.

10 (i) $$\bar{A}^1_{40:50} = \int_0^\infty v^t \, {}_tp_{40:50}\mu_{40+t}dt = .04\int_0^\infty e^{-(.04+.06+\ln 1.05)t}dt = .04\int_0^\infty e^{-0.14879t}dt$$

$$= .04\left[-\frac{e^{-.14879t}}{.14879}\right]_0^\infty = \frac{.04}{.14879} = 0.26884$$

(ii) $$\bar{a}_{40:50:\overline{20}|} = \int_0^{20} v^t \, {}_tp_{40:50}dt = \int_0^{20} e^{-.14879t}dt$$

$$= \left[-\frac{e^{-0.14879t}}{0.14879}\right]_0^{20} = \frac{1}{0.14879}(1 - e^{-2.976}) = 6.378$$

$$\bar{a}_{40:50:\overline{30}|} = \int_0^{30} v^t \, {}_t p_{40:50} dt = \int_0^{30} e^{-.14879t} dt$$

$$= \left[-\frac{e^{-0.14879t}}{0.14879} \right]_0^{30} = \frac{1}{0.14879}(1 - e^{-4.464}) = 6.643$$

Let Premium = *P*, then

$$P(0.75 \times 6.643 + .25 \times 6.378) = 75,000 \times 0.26884$$

$$P = \frac{20163}{6.577} \Rightarrow P = 3065.7$$

Generally part (i) was done well but part (ii) was poorly done. A large proportion of students did not appreciate how to derived the premium relationship described in the question. Another common error was to take the force of interest as 5% rather than ln(1.05)%.

11 The annuity can be written as (with 65 denoting the male life and 62 the female):

$$50000 a^{(12)}_{\overline{65:62}} + 25000 a^{(12)}_{65:62} + 25000 a^{(12)}_{65} + 20000(v^{10} \, {}_{10} p_{\overline{65:62}} + v^{20} \, {}_{20} p_{\overline{65:62}})$$

$$a^{(12)}_{65} = \ddot{a}_{65} - \frac{13}{24} = 13.666 - \frac{13}{24} = 13.124$$

$$a^{(12)}_{65:62} = \ddot{a}_{65:62} - \frac{13}{24} = 12.427 - \frac{13}{24} = 11.885$$

$$a^{(12)}_{\overline{65:62}} = \ddot{a}_{65} + \ddot{a}_{62} - \ddot{a}_{65:62} - \frac{13}{24} = 13.666 + 15.963 - 12.427 - \frac{13}{24} = 16.660$$

$$v^{10} \, {}_{10} p_{\overline{65:62}} = \frac{1 - (1 - l_{75}/l_{65})(1 - l_{72}/l_{62})}{(1.04)^{10}}$$

$$= \frac{1 - (1 - 8405.16/9647.797)(1 - 9193.86/9804.173)}{1.48024}$$

$$= 0.67015$$

$$v^{20}\,{}_{20}p_{\overline{65:62}} = \frac{1-(1-l_{85}/l_{65})(1-l_{82}/l_{62})}{(1.04)^{20}}$$

$$= \frac{1-(1-4892.878/9647.797)(1-7147.965/9804.173)}{2.19112}$$

$$= 0.39545$$

So value is:

(50000*16.660)+(25000*11.885)+(25000*13.124)+20000*(.67015+.39545)

= 1479537

Other formulae approaches credited. Also the final answer is very sensitive to rounding and full credit was given to +/–00 to the answer.

Many students found difficulty in reproducing the correct annuities to make up the total value.

12 Let *P* be the monthly premium. Then:

EPV of premiums:

$$12P\ddot{a}^{(12)}_{[40]:\overline{25}|}@6\% = 155.1272P$$

where

$$\ddot{a}^{(12)}_{[40]:\overline{25}|} = \ddot{a}_{[40]:\overline{25}|} - \frac{11}{24}\left(1-{}_{25}p_{[40]}v^{25}\right)$$

$$= 13.290 - \frac{11}{24}\left(1-\frac{8821.2612}{9854.3036}\times 0.233\right) = 12.9273$$

EPV of benefits:

$$72,750\bar{A}^{\,1}_{[40]:\overline{25}|} + 2250\left(I\bar{A}\right)^{1}_{[40]:\overline{25}|} + 131,250A_{[40]:\overline{25}|}^{\,\frac{1}{}} \quad @6\%$$
$$= 72,750\times 0.04032 + 2250\times 0.62876 + 131,250\times 0.208574$$
$$= 2,933.561 + 1,414.71 + 27,375.3375 = 31,723.609$$

where

$$\bar{A}^{1}_{[40]:\overline{25}|} = 1.06^{0.5} A^{1}_{[40]:\overline{25}|} = 1.06^{0.5}\left[A_{[40]:\overline{25}|} - v^{25}\,{}_{25}p_{[40]} \right]$$

$$= 1.06^{0.5}\left[0.24774 - 0.233 \times \frac{8821.2612}{9854.3036} \right] = 0.04032$$

$$\left(I\bar{A}\right)^{1}_{[40]:\overline{25}|} = 1.06^{0.5}\left(IA\right)^{1}_{[40]:\overline{25}|} = 1.06^{0.5}\left[\left(IA\right)_{[40]} - v^{25}\,{}_{25}p_{[40]}\left(25A_{65} + \left(IA\right)_{65}\right) \right]$$

$$= 1.06^{0.5}\left[3.85489 - 0.208574 \times \left(25 \times 0.40177 + 5.50985\right) \right] = 0.62876$$

EPV of expenses:

$$= 1.15P + 210 + 0.025 \times 12 \times P\ddot{a}^{(12)}_{[40]:\overline{25}|} - 0.025P + 85\left[\ddot{a}^{@i'}_{[40]:\overline{25}|} - 1 \right]$$
$$= 1.15P + 210 + 0.025 \times 12 \times P \times 12.9273 - 0.025P + 85 \times \left[15.887 - 1\right]$$
$$= 5.00319P + 1,475.395$$

where

$$i' = \frac{1.06}{1+b} - 1 = 0.04$$

Equation of value gives:

$$155.1272P = 31,723.609 + 5.00319P + 1475.395$$

$$\Rightarrow P = \frac{33,199.00}{150.1240} = £221.14$$

Well prepared students completed this question satisfactorily. Others found difficulty in deriving in particular the expense values. Credit was given in part to the correct approach even if the final arithmetic proved to be inaccurate.

13 (i) Let P be the net annual premium. Then:

EPV of premiums:

$$P\ddot{a}_{45:\overline{15}|} = 11.386P$$

EPV of benefits:

$$60000\bar{A}^{1}_{45:\overline{10}|} + 40000\bar{A}^{1}_{45:\overline{15}|}$$

$$= 60000\frac{i}{\delta}\left(A_{45} - v^{10}\,{}_{10}p_{45}A_{55}\right) + 40000\frac{i}{\delta}\left(A_{45} - v^{15}\,{}_{15}p_{45}A_{60}\right)$$

$$= 60000\frac{0.04}{0.039221}\left(0.27605 - 1.04^{-10}\frac{9557.8179}{9801.3123}0.38950\right)$$

$$+ 40000\frac{0.04}{0.039221}\left(0.27605 - 1.04^{-15}\frac{9287.2164}{9801.3123}0.45640\right)$$

$$= 1190.567 + 1465.406 = £2655.973$$

Equation of value gives:

$$P = \frac{2655.973}{11.386} = £233.27$$

(ii) The net premium reserve at 31.12.13 is given by:

$${}_{10}V_{45:\overline{15}|} = 40000\bar{A}^{1}_{55:\overline{5}|} - 233.27\ddot{a}_{55:\overline{5}|} = 40000\frac{i}{\delta}\left(A_{55} - v^{5}\,{}_{5}p_{55}A_{60}\right) - 233.27 \times 4.585$$

$$= 40000\frac{0.04}{0.039221}\left(0.38950 - 1.04^{-5}\frac{9287.2164}{9557.8179}0.4564\right) - 233.27 \times 4.585$$

$$= 1019.53 - 1069.54 = -£50.01$$

(iii) Explanation:

Policyholder "in debt" at time 10 (with size of debt equal to the negative reserve) as more life cover provided in the first 10 years than is paid for by the level premiums in those years.

Disadvantages:

If policy is lapsed during first ten years (possibly longer) the company will suffer a loss.

Not possible to recover this loss from policyholder.

Possible alterations:

Collect the premiums more quickly e.g. shorten premium paying term, make premiums larger in earlier years, smaller in later years.

Change the pattern of benefits to reduce benefits in first ten years and increase them in last five years.

(iv) During 2013, we have:

Death strain at risk = $100{,}000\ (1.04)^{1/2} + 50.01 = 102{,}030.40$

EDS = $2878 q_{54} \times 102{,}030.40 = 2878 \times 0.003976 \times 102{,}030.40 = 1{,}167{,}526.52$

ADS = $12 \times 102{,}030.40 = 1{,}224{,}364.80$

Mortality profit = $1{,}167{,}526.52 - 1{,}224{,}364.80$ = –£56,838.28 (i.e. a loss)

Question done well for students who had prepared. Common errors were in (ii) where immediate payment on death not computed and not getting the profit correct in (iv). Students were given reasonable credit if they showed understanding of the problem even if all arithmetical calculations not correct.

14 (i) Multiple decrement table:

x	q_x^d	q_x^s
58	0.004649	0.1
59	0.006929	0.1
60	0.008022	0.1

x	$(aq)_x^d$	$(aq)_x^s$	(ap)	$_{t-1}(ap)$
58	0.004649	0.09954	0.895816	1.000000
59	0.006929	0.09931	0.893764	0.895816
60	0.008022	0.09920	0.892780	0.800648

Unit fund (per policy at start of year)

	Yr 1	*Yr 2*	*Yr 3*
value of units at start of year	0.00	2206.33	5072.05
allocation	2250.00	2850.00	3450.00
B/O spread	112.50	142.50	172.50
interest	85.50	196.55	333.98
management charge	16.67	38.33	65.12
value of units at end of year	2206.33	5072.05	8618.41

Cash flows (per policy at start of year)

	Yr 1	*Yr 2*	*Yr 3*
unallocated premium	750.00	150.00	–450.00
B/O spread	112.50	142.50	172.50
expenses	425.00	130.00	130.00
interest	8.75	3.25	–8.15
management charge	16.67	38.33	65.12
extra death benefit	31.58	27.22	3.06
profit vector	431.34	176.86	–353.59

profit vector	431.34	176.86	–353.59
probability in force	1.0	0.895816	0.800648
profit signature	431.34	158.43	–283.10
discount factor	0.943396	0.889996	0.839619
expected p.v. of profit	406.92	141.01	–237.69

Total NPV of expected profit = 310.24

	Yr 1	*Yr 2*	*Yr 3*
premium signature	3000.00	2687.45	2401.94
discount factor	1.0	0.943396	0.889996
expected p.v. of premiums	3000.00	2535.33	2137.72

Total PV of premiums = 7673.05

Profit margin = $\dfrac{310.24}{7673.05} = 4.04\%$

(ii) To calculate the expected provisions at the end of each year we have (utilising the end of year cash flow figures and decrement tables in (i) above):

$$ {}_2V = \frac{353.59}{1.02} = 346.66 $$

$$ {}_1V \times 1.02 - (ap)_{59} \times {}_2V = -176.86 \Rightarrow {}_1V = 130.36 $$

The revised cash flow for year 1 will become:

$$ 431.34 - (ap)_{58} \times {}_1V = 314.56 $$

Hence the table below can now be completed for the revised net present value of expected profit.

	Yr 1	*Yr 2*	*Yr 3*
revised end of year cash flow	314.56	0	0
probability in force	1	0.895816	0.800648
discount factor	0.943396	0.889996	0.839619
expected p.v. of profit	296.76		

$$\text{Profit margin} = \frac{296.76}{7673.05} = 3.87\%$$

Question again done well by students properly prepared. Part (ii) gave more issues as many students could not seem to remember the zeroisation procedure.

Again reasonable credit given for understanding the process where computational errors had occurred.

END OF EXAMINERS' REPORT

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

8 October 2015 (pm)

Subject CT5 – Contingencies
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Calculate:

(a) ${}_{25}p_{40}$

(b) ${}_{10|}q_{[53]}$

(c) $\overline{a}_{55:\overline{10}|}$

Basis:

Mortality AM92
Interest 4% per annum [3]

2 Derive (to the nearest integer) the median of the complete future lifetime of a person aged 30 exact who is subject to the force of mortality shown below:

$$\mu_{30+t} = \begin{cases} .01 & 0 \le t < 10 \\ .02 & 10 \le t < 20 \\ .03 & 20 \le t \end{cases}$$

[3]

3 (i) Describe the difference between an overhead expense and a direct expense in the context of calculating premiums for a life assurance policy. [1]

(ii) (a) State an example of an overhead expense and a direct expense.

(b) Describe the manner in which each example in (a) is usually allowed for in the calculation of premiums. [3]
[Total 4]

4 (i) Describe how education may affect mortality. [1]

(ii) List three examples of the influence of education on mortality. [3]
[Total 4]

5 A special annuity pays 5,000 per annum for five years increasing to 6,000 per annum for the next five years and increasing further to 7,000 thereafter. The payments for the first five years are guaranteed and thereafter are contingent on survival. The annuity is payable monthly in advance.

Calculate the expected present value of this annuity for a life aged 60 exact. Show all your workings.

Basis:

Mortality PMA92C20
Interest 4% per annum

[5]

6 The employees of a manufacturing company are subject to two modes of decrement, mortality and withdrawal from employment.

The independent forces of mortality and withdrawal for employees aged 50 and 51 are given in the following table:

Age	μ_x^d	μ_x^w
50	0.0010	0.15
51	0.0015	0.10

Calculate, showing all your workings, the probability that a new employee aged 50 exact will die as an employee at age 51 last birthday. State any assumptions that you make. [5]

7 A critical illness scheme provides a benefit of 100,000 on death or earlier diagnosis of a critical illness.

(i) Draw and label the appropriate transition diagram. [3]

(ii) Set out an expression for the expected present value of this benefit. [3]

[Total 6]

8 Calculate, showing all your workings, $a_{73.25}^{(4)}$.

Basis:

Mortality PFA92C20 (assume that the force of mortality is constant between ages 73 and 74 only)
Interest 4% per annum [7]

9 A pension scheme provides a pension of one-sixtieth of final pensionable salary on retirement, due to age or ill-health, for each year of service (part years included). Final pensionable salary is average salary over the three years before retirement. Normal retirement age is 65. Members contribute 5% of pensionable salary each year.

(i) Calculate the expected present value of the combined past and future benefits for a member aged 45 exact with 10 years of past service and salary in the previous year of 25,000. [5]

(ii) Calculate the present value of the member's future contributions. [2]

Basis:

Pension Scheme Table in the Formulae and Tables for Examinations

[Total 7]

10 (i) Define in words the Area Comparability Factor. [2]

The table below shows an extract from a study of mortality for Country A and Area N:

	Country A		*Area N*	
Age	*Population*	*Number of deaths*	*Population*	*Number of deaths*
60	100,235	566	25,366	125
61	95,666	621	22,159	121
62	92,386	635	21,864	135

(ii) Calculate, showing all your workings, the Area Comparability Factor for Area N using Country A as the standard population. [3]

(iii) Calculate, showing all your workings, the directly standardised mortality rate for Area N. [2]

[Total 7]

11 An assurance policy provides a benefit of 10,000 payable immediately on the death of the last survivor of a male life aged 55 exact and a female life aged 50 exact.

(i) Calculate, showing all your workings, the expected present value for this policy. [5]

(ii) Derive an expression for the variance of the value of this policy. [3]

Basis:

Mortality PFA92C20
Interest 4% per annum

[Total 8]

12 A life insurance company issues a two year unit-linked endowment assurance policy to a male life aged 45 exact. Level premiums of 6,000 per annum are payable yearly in advance throughout the term of the policy or until earlier death with 98% of each premium being allocated to units. A policy fee of 50 is deducted from the bid value of units at the start of each policy year. The units are subject to a bid-offer spread of 6%. An annual management charge of 1.25% of the bid value of units is deducted at the end of each policy year.

If the policyholder dies during the term of the policy, the death benefit of 200% of the bid value of the units is payable at the end of the policy year of death.

The policyholder may only surrender the policy at the end of the first policy year. On surrender, the bid value of units less a surrender value penalty of 500 is payable.

On maturity, 100% of the bid value of the units is payable.

Management charges are deducted from the unit fund before death, surrender and maturity benefits are paid.

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund	5.0% per annum in year 1 4.5% per annum in year 2
Rate of interest on non-unit fund cash flows	3.0% per annum in both years 1 & 2
Mortality	AM92 Select
Surrenders	2.5% of all policies in force at the end of policy year 1
Initial expense	225
Renewal expense	80 on the second premium date
Initial commission	7.5% of first premium
Renewal commission	2.5% of the second premium
Death claim expense	90
Maturity claim expense	55
Risk discount rate	6% per annum

(i) Calculate, showing all your workings, the non-unit fund cash flows in the first and second years of the policy if the policyholder:

(a) dies in the first year of the policy.
(b) surrenders in the first year of the policy.
(c) dies in the second year of the policy.
(d) survives to the end of the policy.

[7]

(ii) Derive the expected present value of profit for the policy in the event that the policyholder:

(a) dies in the first year of the policy.
(b) surrenders in the first year of the policy.
(c) dies in the second year of the policy.
(d) survives to the end of the policy.

[5]

(iii) Calculate, showing all your workings, the expected present value of the profit for the policy. [1]

[Total 13]

13 A life assurance company issues a policy to a male life aged 40 exact which provides the following benefits:

- An annuity of 30,000 per annum, payable annually in advance starting on the policyholder's 65^{th} birthday and continuing for life thereafter. The annuity increases by 1,500 each year, with the first increase given on the policyholder's 66^{th} birthday.

- A decreasing term assurance with a death benefit, payable immediately on death, which is given by the formula:

$$10,000 \times (25 - t) \qquad t = 0, 1, 2, \ldots, 24$$

where t denotes the curtate duration in years since inception of the policy. Death benefit cover ceases at age 65.

The policy is paid for by level monthly premiums payable in advance from the date of issue for 25 years, but ceasing on earlier death.

The company uses the following premium basis for the policy:

Mortality	AM92 Select
Interest	4% per annum
Initial commission	35% of the total premiums payable in the first policy year
Initial expenses	225
Renewal commission	5% of the second and subsequent monthly premiums
Renewal expense	55 per annum at the start of the second and subsequent policy years
Death benefit claim expense	275
Annuity payment expense	2.5% of each annuity payment

The renewal expense and the death benefit claim expense are both assumed to increase continuously at 4% compound per annum from inception of the policy and to cease at age 65, or earlier death.

Calculate, showing all your workings, the monthly premium for the policy. [13]

14 (i) Write down in the form of symbols, and also describe, the expression "death strain at risk". [2]

On 1 January 2011, a life insurance company issued the following three types of policies to male lives aged 55 exact:

A 5-year pure endowment assurances with a sum assured of 75,000.

B 5-year term assurances with a sum assured of 75,000, where the death benefit is payable at the end of the year of death.

C 5-year single premium temporary immediate annuities with an annual benefit payable in arrear of 15,000.

For policies A and B, premiums are payable annually in advance throughout the policy term or until earlier death.

(ii) Calculate, showing all your workings, the death strain at risk for each type of policy during 2014. [8]

Basis:

Mortality	AM92 Ultimate for policies A and B PMA92C20 for policy C
Expenses	Ignore
Interest	4% per annum

At the beginning of 2014, the numbers of policies in force were:

Pure endowment assurances	984
Term assurances	3,950
Temporary immediate annuities	495

During 2014, the actual deaths were 5 from policy A, 22 from policy B and 2 from policy C.

(iii) Calculate, showing all your workings, the total mortality profit or loss to the company for 2014 using the same basis as in (ii). [5]

[Total 15]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2015

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

F Layton
Chairman of the Board of Examiners
December 2015

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Contingencies subject is to provide a grounding in the mathematical techniques which can be used to model and value cashflows dependent on death, survival, or other uncertain risks.

2. CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions actuarial work.

3. Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given where appropriate valid points are made which do not appear in the solutions below.

4. In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

5. Students should note that for long questions reasonable credit is given if they can describe the right procedures although to score high marks reasonable accurate numerical calculation is necessary.

B. General comments on *student performance in this diet of the examination*

1. The general performance was lower in this session than the exceptionally high result of the April 2015 examination although reasonably in line with earlier sessions.

2. Well prepared students on the whole did very well in this paper in most questions. In general the questions that were done less well were 2, 8 and 12. The examiners hope that the detailed solutions given below will assist students with further revision.

3. Most of the short questions were very straightforward where an answer could be produced quickly and this is where many successful candidates scored particularly well.

4. It is worth repeating that reasonable credit was given if a student could demonstrate on the longer questions that they understood the processes required even if not all computations were accurate.

C. Comparative pass rates for the past 3 years for this diet of examination

Year	*%*
September 2015	51
April 2015	59
September 2014	52
April 2014	52
September 2013	56
April 2013	53

Reasons for any significant change in pass rates in current diet to those in the past:

See B. above.

Generally this paper was deemed to be a similar standard as those in the past except for April 2015 which students found more straightforward than anticipated. Otherwise there is reasonable consistency.

September 2015 was a little lower because of very poor experience in certain overseas centres (others performed to high standard).

Solutions

Q1 (a) $${}_{25}p_{40} = l_{65} / l_{40} = 8821.2612 / 9856.2863 = 0.894988$$

(b) $${}_{10|}q_{[53]} = d_{63} / l_{[53]} = 102.5202 / 9621.1006 = 0.010656$$

(c) $$\overline{a}_{55:\overline{10}|} = (\ddot{a}_{55} - 0.5 - v^{10}(l_{65} / l_{55})(\ddot{a}_{65} - 0.5))$$

$$= 15.873 - 0.5 - 0.67556 \times \left(\frac{8821.2612}{9557.8179}\right) \times (12.276 - 0.5)$$

$$= 8.031$$

(a) and (b) were done well.

There was a surprising poor showing on (c) where the most common error was to assume the answer was to deduct 0.5 from a 10 year life annuity due

Q2 $\Pr(T_x \le n) = 0.5$

Therefore $e^{-.01*10} \times e^{-.02*10} \times e^{-.03*(n-20)} = 0.5$

$\therefore e^{0.3-0.03n} = .5$

$\Rightarrow -0.3 + 0.03n = -\ln(0.5) = 0.69315$

$\Rightarrow n = \dfrac{0.99315}{.03} = 33.11$

So the total median future lifetime is 33 to nearest whole year

A very simple question which was poorly done overall. Many students did not appear to know how to start the question.

Q3 (i) An overhead expense is an expense that <u>does not</u> vary with the amount of business written

A direct expense is an expense that <u>does</u> vary with the amount of business written

(ii) (a) <u>Overhead Expense</u>

Central services e.g. premises, IT, legal (allowed for on a per policy per annum basis with allowance for inflation)

<u>Direct Expense</u>

- Underwriting (allowed for on a per policy basis although medical expenses might be sum assured related)
- Processing proposal and issuing policy (allowed for on a per policy basis)
- Initial Commission (allowed for directly and usually premium related)
- Renewal Administration (allowed for on a per policy per annum basis with allowance for inflation)
- Renewal Commission (allowed for directly and usually premium related)

(b) See (a) for how expenses are allowed for (shown in brackets).

Generally well done. All reasonable descriptions were credited

Q4 (i) Education influences awareness of healthy lifestyle that reduces morbidity and hence mortality

Education includes formal education and public health campaigns

(ii) This manifests itself through many proximate determinants:

- Increased income
- Better diet choices
- Exercise
- Health care
- Moderation in alcohol consumption or smoking
- Awareness of dangers of drug abuse
- Awareness of safe sexual lifestyle

Generally well done. All reasonable descriptions were credited.

Q5 $\text{EPV} = 5000\ddot{a}^{(12)}_{5|} + 6000\,{}_{5|}\ddot{a}^{(12)}_{60} + 1000\,{}_{10|}\ddot{a}^{(12)}_{60}$

$$= 5000\times\left(\frac{1-v^5}{d^{(12)}}\right) + 6000\times v^5\,{}_5p_{60}\left(\ddot{a}_{65} - 11/24\right) + 1000\times v^{10}\,{}_{10}p_{60}\left(\ddot{a}_{70} - 11/24\right)$$

$$= 5000\times\left(\frac{1-v^5}{0.039157}\right) + 6000v^5\times\frac{9647.797}{9826.131}\times\left(13.666 - \frac{11}{24}\right)$$

$$+1000v^{10}\times\frac{9238.134}{9826.131}\times\left(11.562 - \frac{11}{24}\right)$$

$$= 22738.32 + 63952.31 + 7052.36$$

$$= 93743 \text{ rounded}$$

Generally well done. However many students gave themselves considerable extra work by valuing a deferred 5 year annuity for 6000 for a 5 year term at 60 and then a deferred 10 year annuity for 7000 at 60. The above approach which relies only on whole life annuities is much easier.

Q6 Using:

$$(aq)^d_{50} = \frac{\mu^d_{50}}{\mu^d_{50}+\mu^w_{50}}\left(1-e^{-(\mu^d_{50}+\mu^w_{50})}\right) = \frac{0.001}{0.151}\left(1-e^{-0.151}\right) = 0.0009282$$

$$(aq)^w_{50} = \frac{\mu^w_{50}}{\mu^d_{50}+\mu^w_{50}}\left(1-e^{-(\mu^d_{50}+\mu^w_{50})}\right) = \frac{0.15}{0.151}\left(1-e^{-0.151}\right) = 0.1392241$$

Construct a multiple decrement table assuming the radix of the table is 100,000 lives.

At age 50:

Number of deaths over year = $100,000 \times (aq)^d_{50} = 92.82$

Number of withdrawals over year = $100,000 \times (aq)^w_{50} = 13,922.41$

Age	*No of lives*	*No of deaths over year*	*No of withdrawals over year*
50	100,000.00	92.82	13,922.41
51	85,984.77		

At age 51:

$$(aq)^d_{51} = \frac{\mu^d_{51}}{\mu^d_{51}+\mu^w_{51}}\left(1-e^{-(\mu^d_{51}+\mu^w_{51})}\right) = \frac{0.0015}{0.1015}\left(1-e^{-0.1015}\right) = 0.0014264$$

Number of deaths over year = $85,984.77 \times (aq)^d_{51} = 122.65$

Probability that a new employee aged 50 exact will die as an employee at age 51 last birthday = 122.65 / 100,000 = 0.00123 i.e. **0.123%**

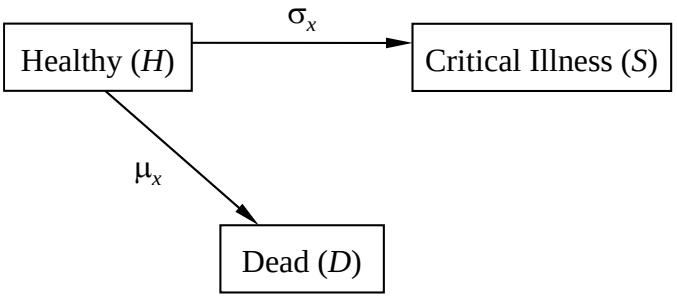
Assumption: The independent forces of mortality and withdrawal are constant over each year of age.

The above solution is a complete analysis. Note the numerical part could also be simply solved directly as follows:

$$(ap)_{50}.(aq)^d_{51} = e^{-(0.001+0.15)}.\frac{0.0015}{(0.1+0.0015)}\left(1-e^{-(0.1+0.0015)}\right) = 0.0012265$$

Reasonably done but many students failed to organise the probabilities properly. Full credit was given for the shorter direct approach as long as the assumptions were also stated.

Q7 Firstly we need the transition model:



The appropriate expression is:

$$EPV = \int_0^{\infty} 100,000\, e^{-\delta t}\, {}_t p_x^{HH} \left(\sigma_{x+t} + \mu_{x+t}\right) dt$$

Or

$$EPV = \int_0^{\infty} 100,000\, e^{-\delta t}\, {}_t(ap)_x \left((a\mu)_{x+t}^{S} + (a\mu)_{x+t}^{D}\right) dt$$

A very easy question generally well done. Students who drew a transition line from (*S)* to (*D)* were penalised as that did not form part of the benefit structure.

Q8 $a_{73.25}^{(4)} = 0.25\,{}_{0.25}p_{73.25} v^{0.25} + 0.25\,{}_{0.5}p_{73.25} v^{0.5} + \ddot{a}_{74}^{(4)}\,{}_{0.75}p_{73.25} v^{0.75}$

Assuming a constant force of mortality between ages 73 and 74 we are required to solve for the constant μ not using μ_{73}

$$p_{73} = 1 - q_{73} = 1 - 0.014973 = 0.985027 = e^{-\mu} \text{ hence } \mu = \ln(.985027) = 0.015086$$

$${}_{0.25}p_{73.25} = e^{-0.25\times 0.015086} = 0.996236$$

$${}_{0.5}p_{73.25} = e^{-0.5\times 0.015086} = 0.992485$$

$${}_{0.75}p_{73.25} = e^{-0.75\times 0.015086} = 0.988749$$

$$\ddot{a}_{74}^{(4)} = \ddot{a}_{74} - \frac{3}{8} = 11.333 - 0.375 = 10.958$$

Hence

$$a^{(4)}_{73.25} = 0.25 \times 0.996236 v^{0.25} + 0.25 \times 0.992485 v^{0.5} + 10.958 \times 0.988749 v^{0.75}$$

$$= 11.011$$

A question which combined a non integer age annuity using a constant force of mortality with a whole life constituent also. The question was very poorly done.

It is also possible to start with $a^{(4)}_{73}$ and deduct off the first quarter.

Q9 (i)

$$\frac{25,000}{60}\left[\frac{10\left({}^{z}M^{ia}_{45} + {}^{z}M^{ra}_{45}\right) + \left({}^{z}\overline{R}^{ia}_{45} + {}^{z}\overline{R}^{ra}_{45}\right)}{s_{44}D_{45}}\right]$$

$$= \frac{25,000}{60}\left[\frac{10\left(52554 + 128026\right) + \left(609826 + 2244130\right)}{8.375 \times 2329}\right]$$

$$= 99,540$$

(ii)

$$5\% \times 25,000 \times \left[\frac{{}^{s}\overline{N}_{45}}{s_{44}D_{45}}\right]$$

$$= 5\% \times 25,000 \times \left[\frac{253080}{8.375 \times 2329}\right]$$

$$= 16,219$$

An easy question generally very well done if students had prepared.

Q10 (i) The area comparability factor is the ratio of the crude mortality rate in a standard population to the crude mortality rate of a sub-population, if that sub-population exhibited standard mortality.

(ii)

	Country A			*Area N*		
Age	*Population*	*Number of deaths*	*Mortality rate*	*Population*	*Actual deaths*	*Expected deaths*
60	100,235	566	0.005647	25,366	125	143
61	95,666	621	0.006491	22,159	121	144
62	92,386	635	0.006873	21,864	135	150
Total	288,287	1,822		69,389	381	437

The area comparability factor = ((1,822/288,287) / (437/69,389)) = 1.0027
(after rounding deaths to 1 decimal place)

(iii) The directly standardised mortality rate for Area N is

(100,235 * 125/25,366 + 95,666 * 121 / 22,159
+ 92,386 * 135 / 21,864) / (100,235 + 95,666 + 92,386) = 0.0055

Note that this question is sensitive to rounding.

Generally straightforward and well done.

Q11 (i) Using the premium conversion relationship:

$$\text{Value} = 10,000 \times \overline{A}_{\overline{55:50}}$$

$$= 10,000 \times (1.04)^{1/2} \times \left(1 - \frac{.04}{1.04} \times \ddot{a}_{\overline{55:50}}\right)$$

$$= 10,000 \times (1.04)^{1/2} \times \left(1 - \frac{.04}{1.04} \times \left(\ddot{a}_{55} + \ddot{a}_{50} - \ddot{a}_{55:50}\right)\right)$$

$$= 10,000 \times (1.04)^{1/2} \times \left(1 - \frac{.04}{1.04} \times (17.364 + 19.539 - 16.602)\right)$$

$$= 2,235$$

(ii) Let the status $u = \overline{x:y}$

Then $\overline{A}_u = \int_0^\infty v^t \mu_{u+t} dt$

The second moment is $\int_0^\infty \left(v^t\right)^2 \mu_{u+t} dt = {}^2\overline{A}_u$

Assuming the two lives are independent then the variance is

$$(10,000)^2\left({}^2\overline{A}_u\right) - (10,000\overline{A}_u)^2$$

Alternatively:

$$\text{var}\left[10,000\, v^{T_{\overline{55:50}}}\right] = 10,000^2\left\{E\left[\left(v^{T_{\overline{55:50}}}\right)^2\right] - \left(E\left[v^{T_{\overline{55:50}}}\right]\right)^2\right\}$$

$$= 10,000^2\left\{E\left[\left(v^2\right)^{T_{\overline{55:50}}}\right] - \left(\overline{A}_{\overline{55:50}}\right)^2\right\}$$

$$= 10,000^2\left\{{}^2\overline{A}_{\overline{55:50}} - \left(\overline{A}_{\overline{55:50}}\right)^2\right\}$$

Part (i) was done well but part (ii) gave difficulties.

It should be noted that there was a small omission in the question wording. The basis should also, of course, have included the male single mortality table. Most students gave the correct answer in any event but any student using female mortality throughout for the single life function was given full credit.

Q12

Annual premium	6000.00	Allocation % (1st yr)	98.0%
Risk discount rate	6.0%	Allocation % (2nd yr)	98.0%
Interest on investments (1st yr)	5.0%	B/O spread	6.0%
Interest on investments (2nd yr)	4.5%	Management charge	1.25%
Interest on non-unit funds (1st and 2nd yrs)	3.0%	Policy Fee	£50
Death benefit (% of bid value of units)	200%		

		% premium
Initial expense/commission	225	7.5%
Renewal expense/commission	80	2.5%
Death claim expense	90	
Maturity claim expense	55	

Mortality table:

x	t	$q^{d}_{[x]+t-1}$	$q^{s}_{[x]+t-1}$	$(aq)^{d}_{[x]+t-1}$	$(aq)^{s}_{[x]+t-1}$	$(ap)_{[x]+t-1}$	$_{t-1}(ap)_{[x]}$
45	1	0.001201	0.02500	0.001201	0.02497	0.973829	1.000000
46	2	0.001557	0.00000	0.001557	0.00000	0.998443	0.973829

Unit fund (per policy at start of year)

	yr 1	*yr 2*
value of units at start of year	0.000	5679.172
alloc	5880.000	5880.000
B/O	–352.80	–352.800
policy fee	–50.000	–50.000
interest	273.860	502.037
management charge	–71.888	–145.730
value of units at year end	5679.172	11512.678

Non Unit fund cash flows (per policy at start of year)

	yr 1	*yr 2*
unallocated premium + policy fee	170.000	170.000
b/o spread	352.800	352.800
expenses	–675.000	–230.000
interest	–4.566	8.784
man charge	71.888	145.730
extra death benefit	-6.821	–17.925
surrender penalty	12.485	0.000
claim expense (death/maturity)	–0.108	–55.054
end of year cash flow	–79.322	374.335

(i) (a) if p/h dies in the 1st year of contract, non unit cash flows at end of the year are:

$yr1 = (170 + 352.80 - 675 - 4.566 + 71.888 - 5679.172 - 90) = -5854.050$

(b) if p/h surrenders in the 1st year of contract, non unit cash flows at end of the year are:

$$yr1 = (170 + 352.80 - 675 - 4.566 + 71.888 + 500) = 415.122$$

(c) if p/h dies in the 2nd year of contract, non unit cash flows at end of each year are:

$$yr1 = (170 + 352.80 - 675 - 4.566 + 71.888) = -84.878$$

$$yr\ 2 = (170 + 352.8 - 230 + 8.784 + 145.73 - 11512.678 - 90) = -11155.364$$

(d) if p/h survives to the end of the contract, non unit cash flows at end of each year are:

$yr\ 1 = -84.878$ (derived above)

$$yr\ 2 = (170 + 352.8 - 230 + 8.784 + 145.73 - 55) = 392.314$$

(ii) (a) if p/h dies in the 1st year of contract, expected present value of profit is given by:

$$-5854.050 \times v \times (aq)^{d}_{[45]} = -5522.689 \times 0.001201 = -6.633$$

(b) if p/h surrenders in the 1st year of contract, expected present value of profit is given by:

$$415.122 \times v \times (aq)^{s}_{[45]} = 391.624 \times 0.02497 = 9.779$$

(c) if p/h dies in the 2nd year of contract, expected present value of profit is given by:

$$\left[-84.878 \times v - 11155.364 \times v^2\right] \times (ap)_{[45]} \times (aq)^{d}_{[45]+1}$$

$$= \left[-80.074 - 9928.234\right] \times 0.973829 \times 0.001557 = -15.175$$

(d) if p/h survives to the end of the contract, expected present value of profit is given by:

$$\left[-84.878 \times v + 392.314 \times v^2\right] \times {}_2(ap)_{[45]}$$

$$= \left[-80.074 + 349.158\right] \times 0.973829 \times 0.998443 = 261.634$$

(iii) Expected present value of the profit of the policy is therefore

= –6.633 + 9.779 – 15.175 + 261.634 = **249.605**

This question proved to be the most difficult on the paper and was in general poorly done. In essence the question was about breaking the final Present Value of Future Profits down into constituent parts which would need to be carried out in any event and each part in itself is relatively straightforward.

Reasonable partial credit was given if a good understanding was shown without the calculations being fully accurate.

Q13 (i) Let P be the monthly premium for the contract. Then:

EPV of premiums is:

$$12P\ddot{a}^{(12)}_{[40]:\overline{25}|} = 12P\left[\ddot{a}_{[40]:\overline{25}|} - \frac{11}{24}\left(1 - v^{25}\,{}_{25}p_{[40]}\right)\right]$$

$$= 12P\left[15.887 - \frac{11}{24}\left(1 - 0.37512 \times \frac{8821.2612}{9854.3036}\right)\right]$$

$$= 186.9909P$$

EPV of death benefits:

$$260{,}000\bar{A}^{1}_{[40]:\overline{25}|} - 10{,}000\left(I\bar{A}\right)^{1}_{[40]:\overline{25}|} = 10{,}000 \times (1.04)^{0.5}\left[26A^{1}_{[40]:\overline{25}|} - (IA)^{1}_{[40]:\overline{25}|}\right]$$

$$= 10198.04\left[26 \times 0.05316 - 0.87602\right] = 5161.64$$

where

$$A^{1}_{[40]:\overline{25}|} = A_{[40]:\overline{25}|} - v^{25}\,{}_{25}p_{[40]} = 0.38896 - 0.33580 = 0.05316$$

and

$$(IA)^{1}_{[40]:\overline{25}|} = (IA)_{[40]} - v^{25}\,{}_{25}p_{[40]}\left[25A_{65} + (IA)_{65}\right]$$

$$= 7.95835 - 0.33580\left[25 \times 0.52786 + 7.89442\right] = 0.87602$$

EPV of annuity:

$$v^{25}\,{}_{25}p_{[40]}\left[28500\ddot{a}_{65}+1500\left(I\ddot{a}\right)_{65}\right]$$

$$=0.33580\left[28500\times12.276+1500\times113.911\right]=174{,}861.97$$

EPV of expenses:

(a) Death claim

$$275\left[1.04^{0.5}q_{[40]}v^{0.5}+1.04^{1.5}\,p_{[40]}q_{[40]+1}v^{1.5}+.....+1.04^{24.5}\,{}_{24}p_{[40]}q_{64}v^{24.5}\right]$$

$$=275\times\,{}_{25}q_{[40]}=275\left(1-0.895168\right)=28.83$$

(b) Annuity

$0.025 \times EPV$ of annuity = 4,371.55

(c) Premium related

$$0.35\times12P+0.05\times12P\left[\ddot{a}^{(12)}_{[40]:\overline{25}|}-\frac{1}{12}\right]=4.2P+0.6P\times\left(15.5826-0.08333\right)$$

$$=13.49956P$$

(d) Other

$$225+55\left(\ddot{a}^{@0\%}_{[40]:\overline{25}|}-1\right)=225+55\left(e_{[40]}-\frac{l_{65}}{l_{[40]}}\left(1+e_{65}\right)\right)$$

$$=225+55\left(39.071-\frac{8821.2612}{9854.3036}\times17.645\right)=1505.17$$

Equation of value gives:

$$186.9909P=5161.64+174861.97+28.83+4371.55$$
$$+13.49956P+1505.17$$

$$\Rightarrow 173.49134P=185929.16$$

$$\Rightarrow P=£1071.69$$

A typical CT5 question, well done by prepared students.

The only real uncertainty was treatment of the death claim expenses.

Again reasonable partial credit was given for understanding without full computational accuracy.

Q14 (i) The death strain at risk for a policy for year $t + 1$ ($t = 0, 1, 2\ldots$) is the excess of the sum assured (i.e. the present value at time $t + 1$ of all benefits payable on death during the year $t + 1$) over the end of year reserve and any benefit payable if the life survives to the end of year $t + 1$.

i.e. DSAR for year $t + 1 = S - \left({}_{t+1}V + R \right)$

(ii) Annual premium for pure endowment with £75,000 sum assured given by:

$$P^{PE} = \frac{75,000}{\ddot{a}_{55:5\rceil}} \times v^5 \times {}_5p_{55} = \frac{75,000}{4.585} \times 0.82193 \times \frac{9287.2164}{9557.8179} = 13,064.223$$

Annual premium for term assurance with £75,000 sum assured given by:

$$P^{TA} = P^{EA} - P^{PE} = \frac{75,000 A_{55:5\rceil}}{\ddot{a}_{55:5\rceil}} - P^{PE}$$

$$= \frac{75,000 \times 0.82365}{4.585} - 13,064.223 = 408.786$$

Reserves at the end of the fourth policy year:

for pure endowment with £75,000 sum assured given by:

$${}_4V^{PE} = 75,000 \times v \times {}_1p_{59} - P^{PE}\ddot{a}_{59:1\rceil}$$

$$= 75,000 \times 0.96154 \times \frac{9287.2164}{9354.0040} - 13,064.223 = 58,536.372$$

for term assurance with £75,000 sum assured given by:

$${}_4V^{TA} = {}_4V^{EA} - {}_4V^{PE}$$

$$= 75,000 A_{59:1\rceil} - (13,064.223 + 408.786)\ddot{a}_{59:1\rceil} - 58,536.372$$

$$= 75,000 \times 0.96154 - (13,064.223 + 408.786) \times 1 - 58,536.372 = 106.119$$

for temporary immediate annuity paying an annual benefit of £15,000 given by:

$$ {}_4V^{IA} = 15,000 a_{59:1\rceil} $$

$$ = 15,000 \times v \times {}_1p_{59} = 15,000 \times 0.96154 \times \frac{9826.131}{9846.908} $$

$$ = 14,392.644 $$

Death strain at risk per policy:

Pure endowment: DSAR = 0 – 58,536.372 = –58,536.372

Term assurance: DSAR = 75,000 – 106.119 = 74,893.881

Immediate annuity: DSAR = 0 – (14,392.644 + 15,000) = –29,392.644

(iii) Mortality profit = EDS – ADS

For pure endowment

$$ EDS = 984 \times q_{58} \times -58,536.372 = 984 \times .006352 \times -58,536.372 = -365,873.866 $$

$$ ADS = 5 \times -58,536.372 = -292,681.86 $$

mortality profit = –73,192.00

For term assurance

$$ EDS = 3950 \times q_{58} \times 74,893.881 = 3950 \times .006352 \times 74,893.881 = 1,879,117.432 $$

$$ ADS = 22 \times 74,893.881 = 1,647,665.382 $$

mortality profit = 231,452.05

For temporary immediate annuity

$$ EDS = 495 \times q_{58} \times -29,392.644 = 495 \times .001814 \times -29,392.644 = -26,391.746 $$

$$ ADS = 2 \times -29,392.644 = -58,785.288 $$

mortality profit = 32,393.54

Hence, total mortality profit = –73,192.00 + 231,452.05 + 32,393.54

= £190,653.59

Another typical CT5 question which was well done by prepared students. Again reasonable partial credit was given for understanding without full computational accuracy.

END OF EXAMINERS' REPORT